

Cyclic Organization in the ‘Chorale’ of Charles Ives’ *Three Quarter-Tone Pieces*

Having a father that was an experimenter, Charles Ives was introduced early to the notion of quarter tones. Later in his life, he turned back to the question of quarter tones, writing the *Three Quarter Pieces* for two pianos in the early 1920’s, based on sketches from the teens, and an essay entitled “Some ‘Quarter-Tone’ Impressions” (Ives 1961). In the essay Ives touches on several issues concerning quarter tones: the ontological status of quarter tones, potential harmonies of the quarter tone system, and the relationship between chord and melody in the quarter-tone system, to name a few.

The third movement of the *Three Quarter-Tone Pieces*, ‘Chorale,’ is unique, differing from the other two pieces in that it is set in a more strict and traditional four-part-harmony style. The setting strongly suggests that Ives was carefully investigating through the ‘Chorale’ the nature and properties of the quartertone system. Many of the harmonies present in the ‘Chorale’ are those that he discusses in his essay and finds to be agreeable. Even though Ives does not recognize it in his essay, the chords that he finds agreeable are those based on cycles of specific generators that are relatively prime to the modulus, 24. The significance of coprime generators has been discussed in recent music theory literature, and it has been shown that these kinds of cycles have special properties, such as the deep scale property and multiplicity.¹ The present paper will focus on the ‘Chorale’ and how these harmonies are treated ‘in action,’ if you will. After presenting a basic formal chart of the piece, the discussion of cyclic organization will begin by taking

¹ See, for example, Clough and Myerson (1985), Carey and Clampitt (1989) and Clough, Engebretsen and Kochavi (1999).

the collections that Ives spells out in his essay and by applying more modern models to show their features. Then it will be argued that a particular seven-note collection (which I call the *sikah baladi* scale) is also in action, even though Ives does not mention it. Then there will be an exploration of the interaction between two particular cycles that are highly cooperative. Finally, it will be shown that one of the sections of the piece is situated in another cyclic universe than the other sections, thus being both contrasting material and a completion of all the cyclic universes based on the possible generators that are relatively prime to 24.

I. Three Quarter-Tone Pieces, 'Chorale'

Before entering into a discussion of the quarter-tone harmonies in the 'Chorale,' I offer a formal diagram of the piece as a whole, given as Figure 1. Measure numbers are given above each subsection. The piece is shown as consisting of four large sections. Sections I and III could almost be considered to be an A and A' of an overall ABA'C form; however, the chord progressions used in the smaller phraselets (mm. 1-9 in section I, and mm. 30-40 in section III) are quite different between these two sections. Nevertheless, sections I and III are similar in that they have a similar basic shape: a section of chord progressions grouped in phraselets, a sequence (which is the same), and a concluding chord progression (which is also the same, labelled as the 'signature' chord progression because of its distinctness).

Section II is motivically and harmonically quite different than the other sections. The dashed bracket at the end of section II indicates that the harmonies used here are based on those used in the other sections, and not on those of the rest of section II. Ives

calls section IV the coda; but rather than trying to determine if it is a coda or not, in the present analysis it will simply be regarded as a distinct section that is also the final section.

At the end of the first three sections there is an instance of the 'signature' chord progression. The second one is slightly different, differing from the other two only by the pentultimate chord. Section IV, on the other hand, ends with a different progression (mm. 58-59) that was first presented in two of the phraslets in section I (mm. 6-7 and repeated in 7-8). This progression features a particular significant melodic line (C–D–E quarter flat).

II. The Two Chords and The Quarter-Tone System²

In his essay on quarter tones, Ives gives a short synopsis of a few of the harmonies of the quarter-tone system that he finds agreeable. His criteria are based in acoustics, as he invokes Helmholtz's notion of beating, as well as in his own personal taste of chord qualities. The main single criterion that he gives is based on the belief that in the quarter-tone system the fifth and the octave are still intervals that assert their authority, being "unrelenting masters in the realm of the physical nature of sound." (112). For this reason he prefers chords that still have pure fifths, rather than fifths either a quarter sharp or a quarter flat;³ though, as will be shown, he does use these intervals. Yet, naturally, chords should also have some quarter-tone intervals, or else they would simply

² The present analysis will use the mod24 system throughout. In the mod24 system, all of the usual mod12 intervals are doubled; for example, i5 in mod12 is equivalent to i10 in mod24, the distance from C to F. All of the even numbers in mod24 correspond to the more familiar mod12 intervals, while the odd numbers correspond to the newly added quarter-tone pitches.

³ In the present essay, the term 'fifth' will always signify, when used in the noun form, that interval that has a 3/2 ratio, and which is expressed in its tempered form as i7 in mod12 and as i14 in mod24. The term 'fourth' is similarly used.

be located in the regular 12-note chromatic universe.

Ives spells out two chords that fit the given criteria, which are shown in Figure 2a. Chord I is a segment of four consecutive notes of the 7-cycle, though Ives does not describe the chord as such. The spelling that he gives begins the 7-cycle upwards starting from C. He describes chord II as "a chord of five notes of equal five-quarter-tone intervals" (113), perhaps recognizing that this chord is the maximally-even distribution of five pitch classes in a mod24 space. Like chord I, chord II is also a segment of consecutive notes in a cycle, though now the 5-cycle. Ives refers to these two chords as the major and the minor chords of the quarter-tone system, further mentioning that they could be expanded to their major and minor correlating scales without suggesting which scales these would be. At another point, Ives calls chord I the 'primary' chord, and chord II the 'secondary' chord. This notion will be discussed later for its invocation of a tonic-dominant polarity.

In Figure 2b, the two chords are laid out in a 2-cycle division—that is, a division between the even and the odd pitch classes. Chord I is built by an intersection of two perfect fifths, one even and one odd. Likewise, chord II is the intersection between an even stack of fourths and an odd stack of fourths, one consisting of three pitches and the other only two. The fact that these chords divide in this way, however, should be of no surprise since $i5$ and $i7$ evenly divide the fourth and the fifth, respectively. Every newly added pitch to each of the cycles adds a new fourth or fifth, and in both cases alternating between the even and odd divisions of the mod24 space. Furthermore, both of these chords are built on cycles whose generators are relatively prime to 24, and thus pass through all 24 pitch classes in the cycle before returning to the starting point. In total,

there are only four interval classes that as generators have this property: 1, 5, 7 and 11.⁴ There is, thus, a fortunate coincidence between 1) Ives' acoustically-based preference for perfect fifths, 2) the fact that i_5 and i_7 are equal divisions of the fourth and the fifth, and 3) the fact that i_5 and i_7 are two of only four interval-class generators that are relatively prime to the modulus, thus yielding collections that have the deep property and multiplicity.

Both of these chords outlined in Figure 2 are prominently featured in the 'Chorale'. While chord II appears in various transpositions, chord I is almost always stated as the pitch classes $\{0,7,14,21\}$. Chord I is the first chord and last chord of the piece, and is used as the final chord in a majority of the phrase endings. In the small phraslets in section I, chord I predominates, though with intermediary uses of chord II. The first nine measures are given as Example 1. As can be seen most of the smaller phraslets (marked by brackets) start and end with chord I (marked by an asterisk).

Chord II never appears as the final chord of a phrase, but appears frequently in the middle of phrases, and often as the penultimate chord of a phrase. Chord II, however, is featured prominently in the two sequential passages and in the final section of the piece. The sequential passages use exclusively 5-cycle chords. Here, the chords move in parallel motion, maintaining identical voicing (see Example 2). The instances of the 5-cycle chord are connected by a single passing tone in the upper-most voice. The sequence

⁴ The number of positive integers that are less than a given modulus m , and relatively prime to m , is given by Euler's function $\phi(m)$. For mod24, $\phi(24) = 8$. The eight integers are 1, 5, 7, 11, 13, 17, 19 and 23. Since complementation partitions these eight numbers into pairs, to find the number of coprime 'interval classes', then, we need only divide $\phi(m)$ by 2. This is always possible since $\phi(m)$ will always be even. Thus for mod24, the number of interval class generators that are coprime with the modulus = $(\phi(24))/2 = 4$.

passes through even intervals, which can be seen, naturally, as mod12 intervals.⁵ In this case, then, the harmonies involve quarter tones, but the intervals of the sequence do not.

Chord II is also strongly featured in section IV. Here, 5-cycle chords plane in parallel motion over a bass pedal C and an ostinato figure (see Example 3). In this example the two upper piano parts are normalized as to more clearly show the parallel motion, placing the fifths in the upper staff and the stacks of fourths in the middle staff. All but a few of the chords that appear over the pedal/ostinato foundation are 5-cycle chords. The chords that are not 5-cycle are in fact 7-cycle—marked with boxes in the example. Ives describes this section in terms of its melody, describing it as being built first on quarter-tone motions, then semitone motions, and finally whole-tone motions. The part of section IV shown in Example 3 centers on the quarter-tone and semitone motions, as can be seen by the interval motions indicated in the example. It is not clear, in fact, what part of the melody Ives was intending to be the whole-step part; one possibility is the last three measures of the piece.

III. The *sikah baladi* Collection

Another prominent collection of the quarter-tone system is presented in Figure 3. If the 7-cycle is allowed to extend past the four notes of chord I, continuing up to seven notes, the collection produced is the maximally-even heptatonic collection. In honor of its existence as a *maqam* of the arabic music tradition, the collection will be referred to as

⁵ $16 \pmod{24} = 8 \pmod{12}$, $20 \pmod{24} = 10 \pmod{12}$, etc.

the *sikah baladi* collection.⁶ Ives may have been suggesting this scale when he mentions in the essay,

“for my father had a weakness for quarter-tones—in fact he didn’t stop even with them. He rigged up a contrivance to stretch 24 or more violin strings and tuned them up to suit the dictates of his own curiosity. He would pick out quarter-tone tunes and try to get the family to sing them, but I remember he gave that up except as a means of punishment—though we got to like some of the tunes *which kept to the usual scale and had some quarter-tones thrown in.*” (110, italics mine)

It is reasonable to assume that the scale that Ives was referring to is the *sikah baladi* scale.⁷ Though there are a plethora of possible seven-note scales that have quarter tones, as testified by the multitude of *maqam* in the classical arabic music tradition, the collection, being an extension of the 7-cycle, has a high concentration of fifths and of neutral thirds,⁸ which would have been attractive to Ives. Furthermore, the collection is maximally-even, which may also have been a concern to Ives, like in the statement, “the chord is a division of the octave,” (112) and in his previously noted description of chord II.

The heptatonic collection can be seen as a readjustment of the usual just-tuned scale, as shown in Figure 3b, with the thirds of the primary triads shifted back a quarter tone. Though the collection could be placed in any of its seven modes (as shown in

⁶ For more on the *maqam* and the intonation of arabic music, see Touma 1996 and Racy 2003, as well as many other texts on arabic music.

⁷ Although it is incorrect to refer to *sikah baladi* as a scale, the name will be used for its simplicity.

⁸ A neutral third is a major third a quarter flat, and lies half way between the major and the minor third.

Appendix A), the collection as shown in Figure 3 was chosen because of the connection with the diatonic scale shown in Figure 3b. Naturally, any of the modes could in fact be chosen and used in composition. Another clue, though, that Ives might have been thinking of this particular mode of the scale, if he was thinking of this scale at all, comes from the ‘Chorale’. The melodic figure that ends the piece has C, D and E quarter flat. Of the various modes, only three have these three pitches as scale degrees 1-2-3. Two of the modes contain an F quarter sharp, and the other contains an F natural. It seems likely that Ives would have gravitated towards the one with the F since it maintains a pure fourth/fifth relationship with the tonic. The mode would be, then, the one as shown in Figure 3.

The first two measures of the ‘Chorale’ reveal the relationship between chord I as being the governing harmony and way the voices unfold. Example 4 shows the first two measures. The phrase (phraslet) begins and ends with chord I. However, the sonorities in between are sonorities that are neither 5-cycle or 7-cycle sonorities. The use of passing tones, such as the Gb in the lower piano, further obscure the underlying progression. If the two piano parts are taken individually, the upper part could be said to be moving within an E quarter-flat major diatonic collection, and the lower part within a C major diatonic collection.⁹ Example 4 shows that the upper and lower piano parts use the same scale degrees within their respective diatonic collections, except that the upper part also includes scale-degree 7. Both parts also use scale-degree #4, used in both cases as a passing tone between scale-degrees 4 and 5.

⁹ Whether the diatonic collections are major or minor is not clear since there is no third or sixth in either collection; however, the difference does not change the outcome of the analysis (as long as the two collections are of the same mode), and for that reason the collections will simply be considered as being two ‘major’ diatonic collections.

The interaction between two diatonic collections an $i7 \pmod{24}$ apart could be read as suggesting an underlying 7-cycle collection of a larger cardinality. Figure 4a shows the cardinality-14 7-cycle collection, here formed by the combination of two diatonic collections. The cardinality-14 set is not significant in itself, but is used more as a intermediary between diatonic collections and *sika baladi* collections. Figure 4b shows that the cardinality-14 collections is not only the composite of two diatonic collections an $i7$ apart, but is also the composite of two *sika baladi* collections an $i1$ apart.¹⁰ This means that *sikah baladi* collections can be created through the interaction of two diatonic collections. Naturally, due to the nature of the instrumentation of the piece, where there are two pianos that are tuned normally to themselves but are tuned a quarter tone apart from each other, the expanded 7-cycle collection is necessary as a compositional resource.

Although Ives never explicitly described such a framework, he does spend some time discussing the comparison (114-5) of overtones between C (256 Hz) and E quarter flat (314 Hz). Ives argues that there is a lack of dissonance between the two tones because the overtones lie well spaced between each other, avoiding disagreeable dissonances caused by conflicting overtones. There is a difference, though, between overtones and how diatonic collections are formed. Diatonic collections are more based on fifths—that is, the fifth is the generator of the system. It is in fact for this reason that the the cardinality14 collection is an intermediary between the two diatonic collections and the two *sikah baladi* collections: $i7 \pmod{24}$, the distance between the two diatonic

¹⁰ The reason that the two *sikah baladi* scales are an $i1$ apart can be explained as follows. Let g be a generator coprime with the modulus, and m the modulus. In the case of $\pmod{24}$, it is true for all g that $g^2 \equiv 1 \pmod{m}$. Thus seven $i7$'s will fall on 1, as will five $i5$'s, and so on. This also means that in the 7-cycle, the heptatonic collection will have ME, and in the 5-cycle, the pentatonic collection will have ME, and so on. However, this does not hold for every modulus.

collections, is also the generator of the fifth. Ives never mentions seeing the fifth as generated by $i7$, however the connection must have been noted on some level since the generators that he is so fond of, $i5$ and $i7$, evenly divide the fourth and the fifth. Nevertheless, the connection between the overtones and the diatonic collection of C and E quarter flat is compelling.

IV. Beyond the *sikah baladi* Scale

Though the *sikah baladi* scale appears in many ways in the ‘Chorale’, as noted before, the piece moves away from the collection quite quickly. Even in the smaller phrases at the beginning of the piece, the voices move to pitches that lie outside of the 7-cycle environment (moving away already in m. 3). Naturally, the analogy of “home, leaving home, and return to home” applies here. Even though not all of the sonorities in the middle of these phrases can be explained as ‘chords’ of a particularly special type, the progressions tend to gravitate to a 5-cycle chord towards the end of the phrase, which in turn moves to the concluding chord I. In the ‘Choral’ there are a few different 5-cycle chords used before the 7-cycle chord. Example 5 shows a few of the instances. In Example 5a, the complete 5-cycle chord is given with the motion to E quarter sharp. The 5-cycle chord and the 7-cycle chord that follows share two common tones, C and G. The same 5-cycle chord minus the E quarter sharp is used again in m. 6, here with the A quarter sharp sustained, acting as a retardation to A three-quarters sharp. Example 5b, from the end of the piece, involves a 5-cycle chord that has only one common tone with the 7-cycle chord that follows. The voice leading between these two chords is quite elegant: one fifth moves up a semitone and the other up a whole tone.

In the two cases of a 5-cycle chord moving to a 7-cycle chord shown in Examples 5a and 5b, C is held as a common tone, and in both cases the C is the lowest pitch and held as a pedal. In another case, however, the 5-cycle chord acts more as a dominant, in that the chord does not contain a C but does share G as a common tone with the 7-cycle chord. The progression is shown as Example 5c. Of the five possible 5-cycle chords that have a G, there are three that also have a C, and two that have a B quarter sharp. This can be seen Example 5d, keeping in mind that each row is an instance of chord II. If one wants to use the 5-cycle chord as a ‘dominant,’ then perforce there are only two options, under the criteria that the chord should have G but not C; and perforce the leading tone will have to be B quarter sharp. The 11-cycle system (see below) also has the leading tone placed a quarter sharp.

Another particular chord not mentioned thus far is also used in the piece. Example 6a shows the concluding chord progression of section IV. The two intervening chords between the two instances of chord I are neither a 5-cycle or a 7-cycle chord. However, they have a fifth and the pitches are well-distributed. Instead of being either 5-cycle or 7-cycle, it is a hybrid of the two. The fifth of the chord, shown as the {0, 14} dyad in Example 6b, is evenly divided by $i7$, and from the other side by $i5$. The chord, as shown in Example 6b, is only a semitone different than chord I. Instead of $pc21$, the hybrid chord has $pc19$. It is also closely related to the 5-cycle chord. In this case, to get all five notes of the 5-cycle chord, $pc7$ of the hybrid chord would have to ‘split.’ In any case, to move to the nearest 5-cycle chord, $pc7$ of the hybrid chord would only have to move either by $i2$ or by $i3$, up or down.

V. Section II and the 11-cycle

Thus far in the paper section II has not been addressed. As mentioned before, not only does section II feature different motivic material, but the harmonies in this section are different from those in the other sections. The shift is abrupt, and harmonically speaking it is the most dissonant section of the piece. The reason for the dissonance is that the pitch material used in this section is what can be described as a ‘mistuned’ diatonic collection (see Example 7a). Salience confirms G as tonic in the section, and thus the collection can be seen as a G major collection with scale-degrees 4,5,6 and 7 sharpened a quarter tone. Unlike the collections so far mentioned, the mistuned G major collection has a paucity of perfect fifths and an abundance of major thirds. In this sense, the collection is antithetical to the 5- and 7-cycles.

The use of the mistuned diatonic collection suggests the third mod24 cycle generated by an interval coprime with the modulus (excluding of course the trivial case of $i1$)—that is, the 11-cycle. Figure 5a gives the 13-note collection generated by $i11$, which is here named the 11-cycle chromatic collection. In the 11-cycle universe, the modular space is divided into two halves at the tritone, one half having all even numbers, and the other odd numbers. Figure 5b shows the set being generated on the circle. With each new added pitch a semitone tone is added, alternating between evens and odds. The mistuned diatonic collection falls into this cycle. There are several ways one could derive a mistuned scale from the 11-cycle chromatic collection, though, but the one that Ives employed has some particular properties. The chord progression at the end of the section, before the ‘signature’ chord progression, shines light on the given mistuning. As shown in Example 7b, the mistuned “I-V-I-IV-I” progression uses two chord types: the “I” chord

has a fifth a quarter sharp, and the “IV” and the “V” have a fifth a quarter flat. All three chords have a major third. The collection chosen allows for primary chords with similar thirds, and more importantly, allows the “IV” and the “V” chords to be of the same set class.

Section II is not only distinctive from the other sections in both motivic material and the underlying harmonies, but the section also completes the array of potential mod24 cyclic systems used in the piece. The section, therefore, acts not only as a contrasting section, but also as the section that explores the other potential realm of cyclically organized quarter tones.

VI. Conclusion

The findings and pieces of advice given by Ives in his essay are strongly realized in the ‘Chorale.’ The connections shown in the present paper between what Ives described in his essay and the ‘Chorale’ seem to suggest that Ives did have in mind some sort of ‘quarter-tone diatonicism,’ setting guidelines for how to conceive of such a system. However, some of the guidelines that he gives could easily be challenged. For example, he treats the 7-cycle and the 5-cycle as the major and the minor of the quarter-tone system, but there is no real justification why it should be this way. The point is, and Ives seems to have discovered this, is that in the quarter-tone system there are three possible generators that generate well-formed collections (i5, i7 and i11) that have special properties (i.e. deep scale property, multiplicity), whereas in the mod12 chromatic universe there is only one, namely i5 (again, excluding the complement, i7). Since in the quarter-tone system there are three such cyclic systems, instead of just one), the

interaction between the three systems must be negotiated. If the ‘Chorale’ is taken as the exemplar, then the 5-cycle system is subordinate to the 7-cycle system, and the 11-cycle system is independent, being a contrasting system to the 5- and 7-cycle systems.

Although it beyond the scope of this project, it would be interesting to see how these ideas play out, if at all, in the other two pieces. However, the strictness of the quarter-tone system as laid out in this paper seems to be more of a rarity in much of the music written that involves quarter tones. A majority of the pieces out there could be more rightly considered as being ‘atonal,’ and not employing this sort of ‘quarter-tone diatonicism.’ As of yet, I know of no other piece besides the ‘Chorale.’

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